Diabologic: Paradox
by Frank Dolinar

"In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead." -- Aristotle, *Physics*

The idea that a fast runner can never overtake a slower runner who has been given a head start is ludicrous to anyone who has ever run -- or simply watched -- a race. Yet this idea, proposed by Zeno of Elea (a Greek philosopher who lived in the 5th century BCE), and the way it was presented caused confusion for over two thousand years. Zeno's paradoxes are so logically reasonable and so counter to experience that they begged for a solution that would resolve the paradox.

The solution was a long time coming.

Zeno's motivation for creating these paradoxes, the so-called "arguments against motion", had its origin in the work of Parmenides of Elea, a contemporary of Zeno. The only known work of Parmenides to have survived is a poem -- or more precisely fragments of the poem. Parmenides describes two views of reality. In the fragment of the poem known as the *Way of Truth*, he presents his view that reality is one, change is impossible, and existence is timeless and uniform. In the fragment known as the *Way of Opinion*, he provides his view on the world of appearances, which is false and deceitful.

While you may not have heard of Parmenides, his ideas strongly influenced Plato, and through him, the philosophic underpinnings of Western civilization.

These paradoxes, particularly the arguments against motion, constituted a major problem for ancient and medieval philosophers, who found most of the attempted solutions of the day to be unsatisfactory.

Here is a more detailed statement of Zeno's paradox of ***Achilles and the Tortoise***:

Achilles (the fastest human runner) is engaged in a footrace with a tortoise. The tortoise is allowed a head start of 100 feet. At some starting signal, each racer begins running at a constant speed (one fast & one slow). After a time, Achilles will have run 100 feet and is now where the tortoise began. But the tortoise has moved on (say 10 feet). It will take Achilles some additional time to reach where the tortoise now is, by which time the tortoise will have moved on again. While each time period gets smaller as Achilles progressively reaches where the tortoise has been, the tortoise has moved on. Therefore, because there are an infinite number of points that Achilles must reach and every time the tortoise has moved on, Achilles can never overtake the tortoise.

This paradox is a study in rate of change and ever smaller measures of distance and time — exactly the kinds of problems dealt with by the calculus, the mid-17th century innovation of mathematics derived (separately but concurrently) by Isaac Newton and Gottfried Wilhelm Leibniz. [Having taken calculus, I'm very glad it uses the notation developed by Leibniz.]

So, in the mid-17th century, mathematicians thought they had done away with Zeno's paradoxes based on solutions derived using the calculus and methods of handling infinite sequences.

The paradoxes pose no problems to engineers, since practical questions of where and when Achilles will actually pass the tortoise are easily handled by the calculus.

But philosophers are not so easy to convince. Some insist that there are deep metaphysical questions raised by Zeno's paradoxes that the calculus does not address, let alone dispel. They do not see how calculus takes anything away from Zeno's reasoning, nor do they see where or how (in terms of the calculus) Zeno's reasoning goes wrong.

Zeno's paradoxes provide solid logical arguments, even though we know from experience that as descriptions of real-world situations their conclusions are simply wrong. But they twist your head around trying to understand where the problem lies, and that's the essence of the paradox.

By the way, one of the best uses of Zeno's Achilles and the Tortoise paradox that I'm aware of occurs in an entertaining, science fiction, short story by Eric Frank Russell titled "Diabologic".